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AN ANALYSIS OF ACCURACY OF A PROCEDURE FOR COMPUTING LOWER CONFIDENCE LIMIT ON SYSTEM RELIABILITY UTILIZING SUBSYSTEM TEST DATA

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THESIS

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ABSTRACT

Systems which are composed of two or more phases, or subsystems, arranged in logical sequence are found frequently in industry and defense. Standard procedures for computing lower confidence limits on reliability of such systems rely on the use of system data. Engineering changes to any of these subsystems can effect the invalidation of all existing data, necessitating additional, sometimes extensive, testing. Such changes are not infrequent in complex systems. A need exists for a method of computing lower confidence limits on reliability which uses phase data. Some approximation techniques have become available. One such technique is currently being used by Applied Physics Laboratory, The Johns Hopkins University. Computer simulation techniques are used to analyze the accuracy of this procedure.

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CHAPTER I

INTRODUCTION

Current technology has produced a class of systems which are composed of two or more phases, or subsystems, arranged in logical sequence, such that the success of the system depends upon the success of the component phases. Thus the reliability of the system, R_s, depends upon the reliability of each of the k phases of which it is composed. We express this mathematically as follows:

$$R_s = \prod_{i=1}^{k} R_i \tag{1}$$

where R_i is the reliability of the i^{th} phase. Standard statistical procedures used to compute lower confidence limits (LCL) on R_s require the use of system test data. Whenever high degrees of reliability, at high confidence levels, are demanded of a system, extensive testing is required, frequently at considerable cost.

Engineering changes to any one of the phases may cause the invalidation of all previous system test data. There is a need for a procedure to construct lower confidence limits on system reliability using phase test data which will salvage as much of the data as possible.

Approximation procedures are becoming available for so constructing these LCL's, one of which is currently being used by Applied Physics Laboratory, The Johns Hopkins University. This procedure is discussed in [1].[1]

The purpose of this paper is to analyze the accuracy of this procedure. Computer simulation techniques will be used to accomplish this analysis.

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[1]Numbers in brackets refer to items in the bibliography with the same number.

CHAPTER II

COMPUTATIONAL PROCEDURE EMPLOYED BY APPLIED PHYSICS LABORATORY

The APL method for constructing LCL's on R_s assumes the following:

a) mutually independent trials

b) the distribution of the reliability estimator, $\hat{R}_s$, is normal.

The formula for computing the 100(1-α)%, one-sided LCL, $\hat{R}_L$, is

$$\hat{R}_L = \hat{R}_s - P(1-\alpha) \cdot \hat{\sigma}_{\hat{R}_s} \tag{2}$$

where $\hat{R}_s$ is the system reliability estimator, $P(1-\alpha)$ is the 100(1-α) percentile of the standard normal distribution, and $\hat{\sigma}_{\hat{R}_s}$ is the standard deviation estimator for the $\hat{R}_s$ distribution

$$\hat{R}_s = \prod_{i=1}^{k} \hat{R}_i \tag{3}$$

where $\hat{R}_i$ is the phase reliability estimator and is defined by

$$\hat{R}_i = s_i/n_i \ . \tag{4}$$

That is, $\hat{R}_i$ denotes the proportion of successes for the i^{th} phase, s_i, out of n_i trials.

The value $\hat{\sigma}_{\hat{R}_s}$ is computed by

$$\hat{\sigma}^2_{\hat{R}_s} = \prod_{i=1}^{k} \hat{R}_i - \prod_{i=1}^{k} \left[\hat{R}_i - \frac{\hat{R}_i(1-\hat{R}_i)}{n_i - 1} \right] \tag{5}$$

which is an unbiased estimator of the variance of $\hat{R}_s$. Standard deviation, $\hat{\sigma}_{\hat{R}_s}$, is obtained by extracting the square root of the variance. Although biased, this estimator is considered to be sufficiently close to $\sigma_{\hat{R}_s}$ [1].

As stated above, one basic assumption of the model is that $\hat{R}_s$ is a normally distributed random variable. As an ancillary result of

the simulation, this assumption was verified for those cases for which the procedure was reasonably accurate. For each case considered, 1000 values of $\hat{R}_s$ were computed and plotted on normal probability paper. For one such example see Figure 1. For those cases which proved less accurate, the distribution departed from normal.

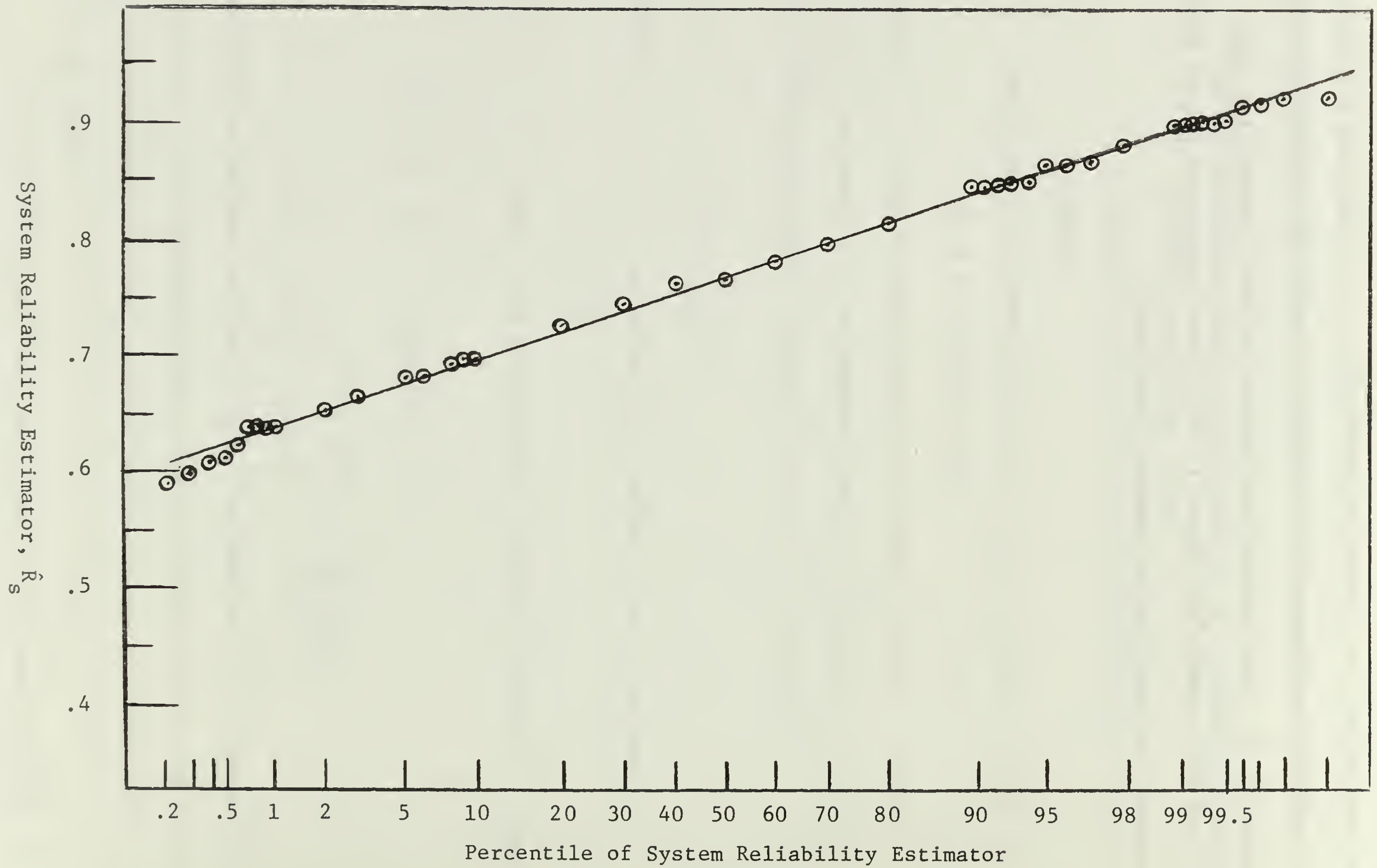

FIGURE 1

ONE EXAMPLE OF THE NORMAL DISTRIBUTION OF SYSTEM RELIABILITY ESTIMATOR (CASE 32, TABLE II)

CHAPTER III

THEORY OF THE SIMULATION TECHNIQUE

One standard interpretation which can be applied to the meaning of a lower confidence limit involves the method used to compute it. This says that regardless of the true value of R_s the computation will produce an LCL, $\hat{R}_L$, which is less than R_s $(1-\alpha)\%$ of the time. Use can be made of this interpretation in formulating the simulation model.

If one were to use an exact method of determining $\hat{R}_L$ and repeated the process several times, he could expect that approximately $(1-\alpha)\%$ of the values would be less than R_s. As the number of replications approaches infinity, he could expect that exactly $(1-\alpha)\%$ of the values would be less than R_s.

One could use a similar approach to evaluate the accuracy of an unproven method. Given a hypothetical system of known R_i, and hence known R_s, one could use the proposed method to compute $\hat{R}_L$ on R_s. If the method was accurate, approximately $(1-\alpha)\%$ of the values so computed would be less than R_s. If the number of replications was sufficiently large, the percentile of $\hat{R}_L$ which is less than R_s would be a good measure of the accuracy of the procedure. This percentile will be denoted $A_{1-\alpha}$

The computer simulation technique, used herein, follows the above approach. The rapid computational ability of the computer allows one to simulate the testing process, compute $\hat{R}_L$, and replicate the process a great many times. One of our measures of effectivess of the method will be the relative closeness of $A_{1-\alpha}$ to R_s. Other measures of effectiveness will be discussed in the results section of this paper.

The simulation technique begins by reading from data the parameters of the case to be considered; i.e., KK, the number of replications desired (1000 for all cases considered herein); K, the number of phases per system; R_i and n_i for all phases; and α. Using a pseudo-random number generator[2] and the known phase reliabilities, test data is gathered on each of the phases by comparing n_i random numbers with R_i $(i = 1, 2, \ldots, K)$.[3] Each time the random number generator produces a number less than or equal to R_i it is counted as a successful test. A running total of successes, s_i, is kept throughout the course of testing. After having so obtained these data, SUBROUTINE RHAT is called with parameter α. The values s_i and n_i are made available to RHAT via the DIMENSION statement. SUBROUTINE RHAT returns the values $\hat{R}_L$ and $\hat{R}_s$. This operation is repeated for KK replications to compute KK values of $\hat{R}_L$ and $\hat{R}_s$. These values are stored in vector matrices for later use.

SUBROUTINE RHAT is a relatively straightforward application of the working formulae contained in Chapter 2. One point worthy of note is that only ten values of $P(1-\alpha)$ have been read into computer memory in order to conserve memory space. The argument $(1-\alpha)$ is digitized to facilitate stowage and retrieval. This limits the values of α which can be used to increments of .05, from .05 to and including .50, a limitation which can easily be changed as the requirements dictate.

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[2]The pseudo-random number generator used herein is a library subroutine of the Computer Facility, Naval Postgraduate School, Monterey. A printout of this subroutine is included in Appendix II.

[3]The reader is invited to follow the program logic by referring to Appendix II. Extensive use is made of comment cards to explain the operations as they are accomplished. The program notation is intended to be the same in meaning as that of the body of the paper. It is, of necessity, somewhat different in form because of machine limitations. As an aid, a glossary of terms is provided in Appendix I.

As the above values of $\hat{R}_L$ are generated, they are summed for eventual division by KK to compute their mean.

Thus far, the simulation has performed the following operations:

1) generated the statistics s_i $(i = 1, 2, 3, \ldots, K)$

2) computed the values for the $\hat{R}_s$, $\hat{R}_L$ and A1 matrices (A1 being an exact duplication of the $\hat{R}_L$ matrix)

3) summed the values of $\hat{R}_L$ to get the mean, $\mu_{\hat{R}_L}$.

Next, a sorting routine, SUBROUTINE SHSORT,[4] is called for the purpose of putting matrices $\hat{R}_s$ and A1 into ascending numerical order. This is done to facilitate plotting these values on probability graph paper to determine their distributions. This also facilitates obtaining the $100(1-\alpha)$ percentile of the $\hat{R}_L$ distribution, $A_{1-\alpha}$. Recall the matrix A1 was identical to the matrix $\hat{R}_L$ before it was ordered. Thus, $A_{1-\alpha}$ equals the $KK(1-\alpha)^{th}$ member of the ordered A1 matrix.

Next, the percentile of $\hat{R}_L$ which is less than R_s is obtained; i.e., the probability $\hat{R}_L$ is less than R_s, denoted $P(R_s)$. This is computed by determining the last element of the ordered A1 matrix whose value is less than or equal to R_s and dividing its index number by KK.

The variance of $\hat{R}_L$, $\sigma_{\hat{R}_L}^2$, is calculated next by summing the squared differences of $\hat{R}_L$ and $\mu_{\hat{R}_L}$ and dividing by KK, the number of replications,

$$\frac{\sum_{i=1}^{KK} (\hat{R}_{L_i} - \mu_{\hat{R}_L})}{KK}$$

[4]This subroutine is a library subroutine of the Computer Facility, Naval Postgraduate School, Monterey, Calif. It is printed out in Appendix II.

The square root is later extracted to obtain the standard deviation, $\sigma_{\hat{R}_L}$.

The difference between R_s and $A_{1-\alpha}$ is then obtained, which is the primary measure of effectiveness of the procedure.

The remaining statements are editorial in nature; e.g., input/output statements which read in data or printout results. Others initialize variables, sums, or products to ensure that values are not carried over from replication to replication.

The program output may be summarized as follows:

a) 1000 values of system reliability estimator, $\hat{R}_s$, in ascending numerical order

b) 1000 values of LCL, $\hat{R}_L$, in chronological order

c) 1000 values of LCL, $\hat{R}_L$, in ascending numerical order

d) system reliability, R_s

e) 100(1-α) percentile of the population of $\hat{R}_L$ population, $A_{1-\alpha}$

f) standard deviation of $\hat{R}_L$ population, $\sigma_{\hat{R}_L}$

g) probability that $\hat{R}_L$ is less than R_s, $P(R_s)$

h) mean of $\hat{R}_L$, $\mu_{\hat{R}_L}$

i) the difference between R_s and $A_{1-\alpha}$

j) $\Sigma n_i q_i$, a measure of the amount of testing done and the unreliability (q_i), or failure rate

CHAPTER IV

SIMULATION RESULTS

The accuracy of the procedure is measured by the closeness of $A_{1-\alpha}$ relative to R_s. A second measure of effectiveness, $P(R_s)$, the probability that $\hat{R}_L$ is less than R_s, provides a measure of the true confidence level achieved by using this procedure. If the method were exact, one could expect $A_{1-\alpha}$ to equal R_s and $P(R_s)$ to equal $(1-\alpha)$.

In addition to the above criteria, the mean of $\hat{R}_L$, $\mu_{\hat{R}_L}$, and the standard deviation, $\sigma_{\hat{R}_L}$, are also provided as measures of effectiveness. These two characteristics provide a measure of the likelihood of the procedure producing a lower confidence limit which differs considerably from R_s. It can be shown that for those cases for which the procedure is reasonably accurate the distribution of $\hat{R}_L$ is approximately normal. (See Figure 2.) Thus if the procedure is accurate, one should expect to find the $1-\alpha$ percentile to be a known function of the mean; e.g., the 90^{th} percentile should be approximately 1.28 standard deviations above the sample mean. In addition, the mean and variance can be used to construct prediction regions into which $\hat{R}_L$ is likely to fall, assuming again that $\hat{R}_L$ is normally distributed.

Several cases were examined in evaluation of this method. In each case one or more of the parameters, K, R_i, n_i, were changed. In addition, each case was run for α-values equal to .05, .1, and .2, successively. The results of these cases are tabulated in Tables I through III.

The accuracy of reliability testing procedures is most often a function of the amount of testing, n_i, and the unreliability of the subsystems, q_i. This quantity is also tabulated in Tables I through

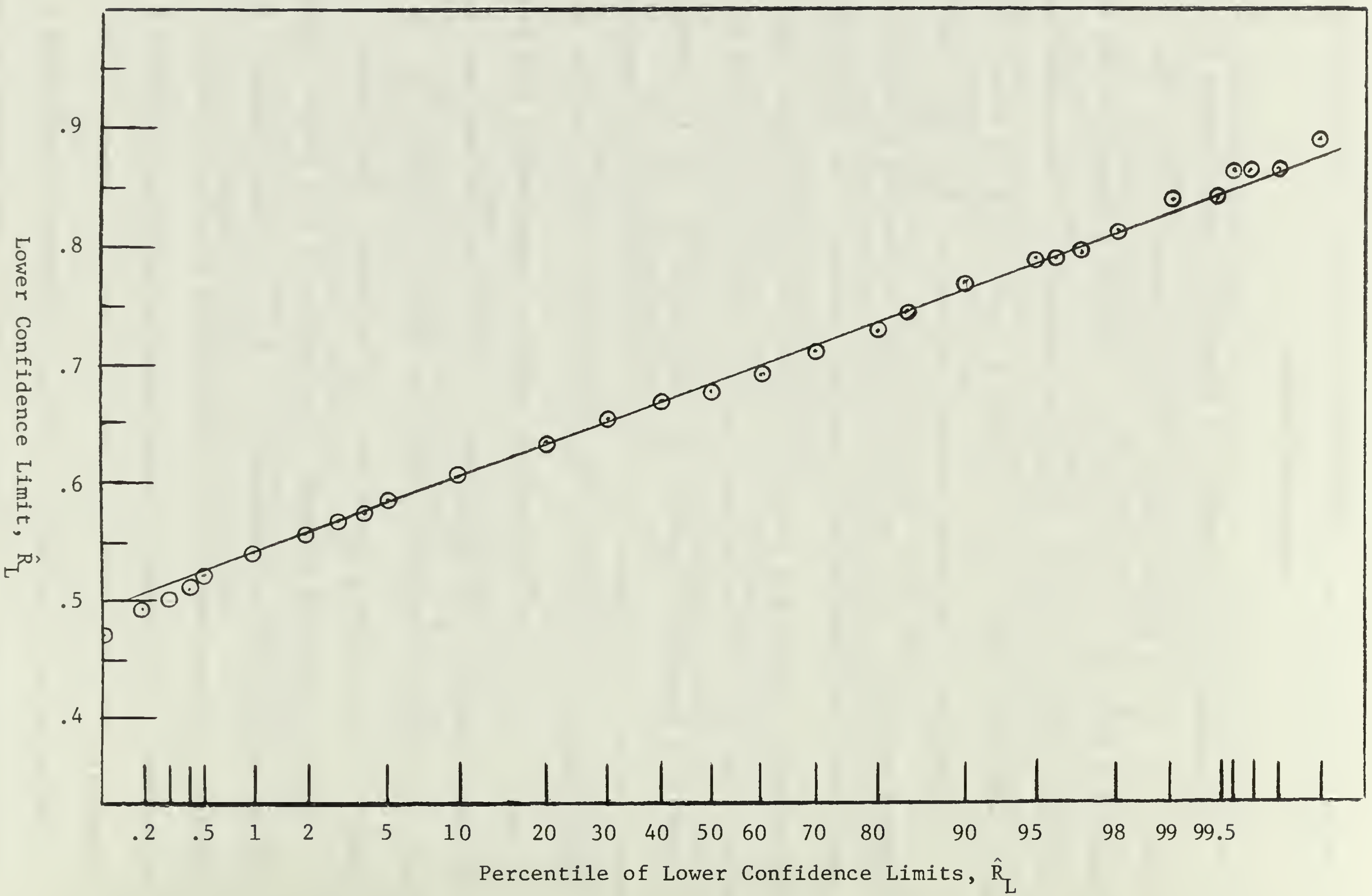

FIGURE 2

ONE EXAMPLE OF THE NORMALITY OF THE DISTRIBUTION, R_L (CASE 32, TABLE II)

III and is expressed by $\sum_{i=1}^{K} n_i q_i$. Thus Tables I through III provide criteria upon which one can judge the accuracy of the procedure and an effective measure of when to expect this accuracy. If one knows the reliabilities for which he is striving and the amount of testing which is economically feasible, he has available a good estimate of the accuracy of this procedure.

The cases considered are tabulated in three separate tables according to the purpose for which they were considered. Cases 1 through 24, Table I, are the same as those considered by Woods and Borsting [2] and are intended to provide a direct comparison with their method. Table III contains a "family" of cases in which only the parameter n_i changes. These cases demonstrate how the accuracy criteria, $A_{1-\alpha}$ and $P(R_s)$, behave as a function of the amount of testing. Figures 3 through 8 graphically illustrate the behavior of the accuracy criteria as a function of $\Sigma n_i q_i$ for the cases considered in Tables I through III. Each of these graphs utilizes one of the three α-levels considered.

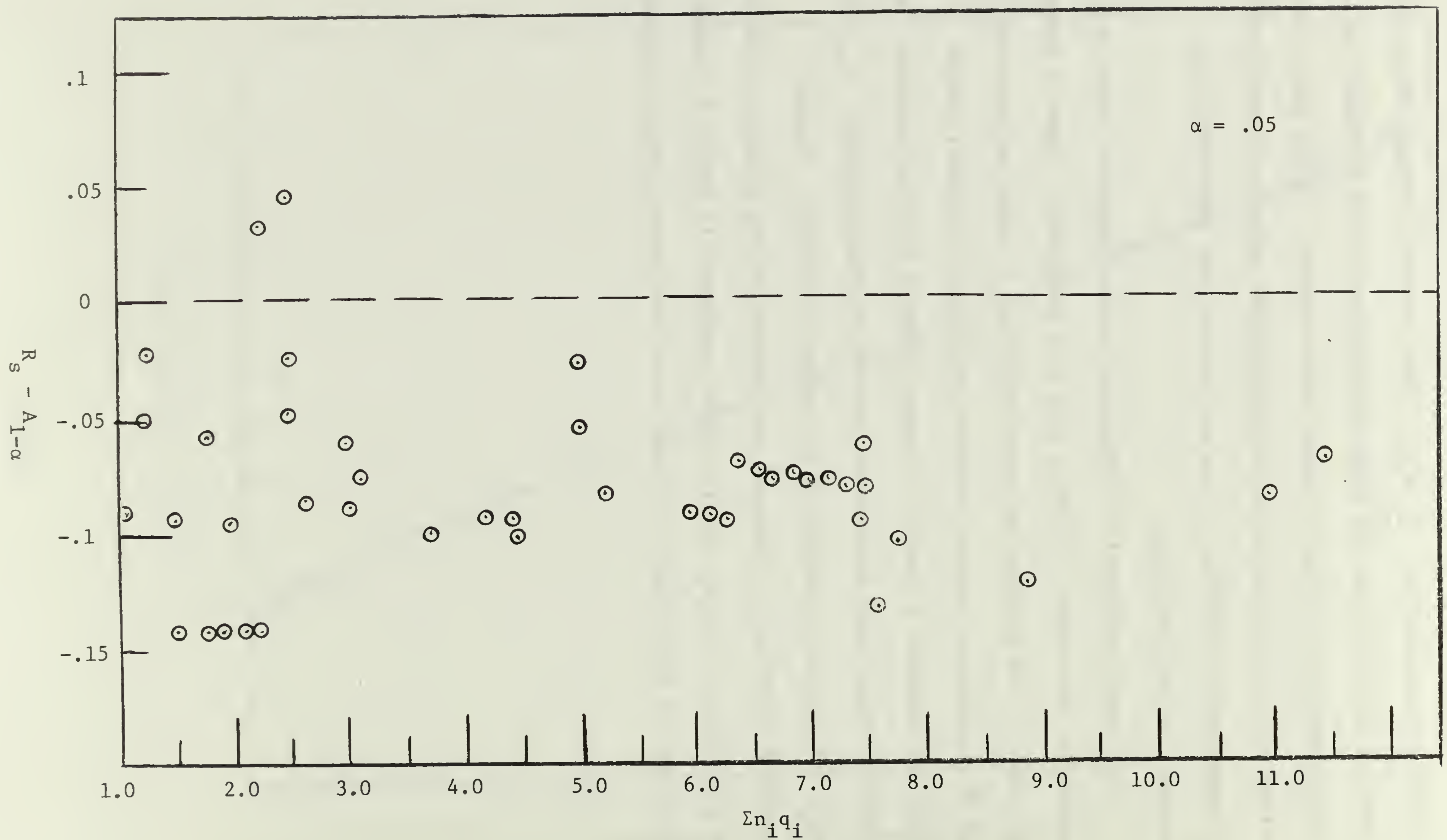

FIGURE 3

MEASURE OF THE ACCURACY CRITERION ($R_s - A_{1-\alpha}$) VERSUS $\Sigma n_i q_i$

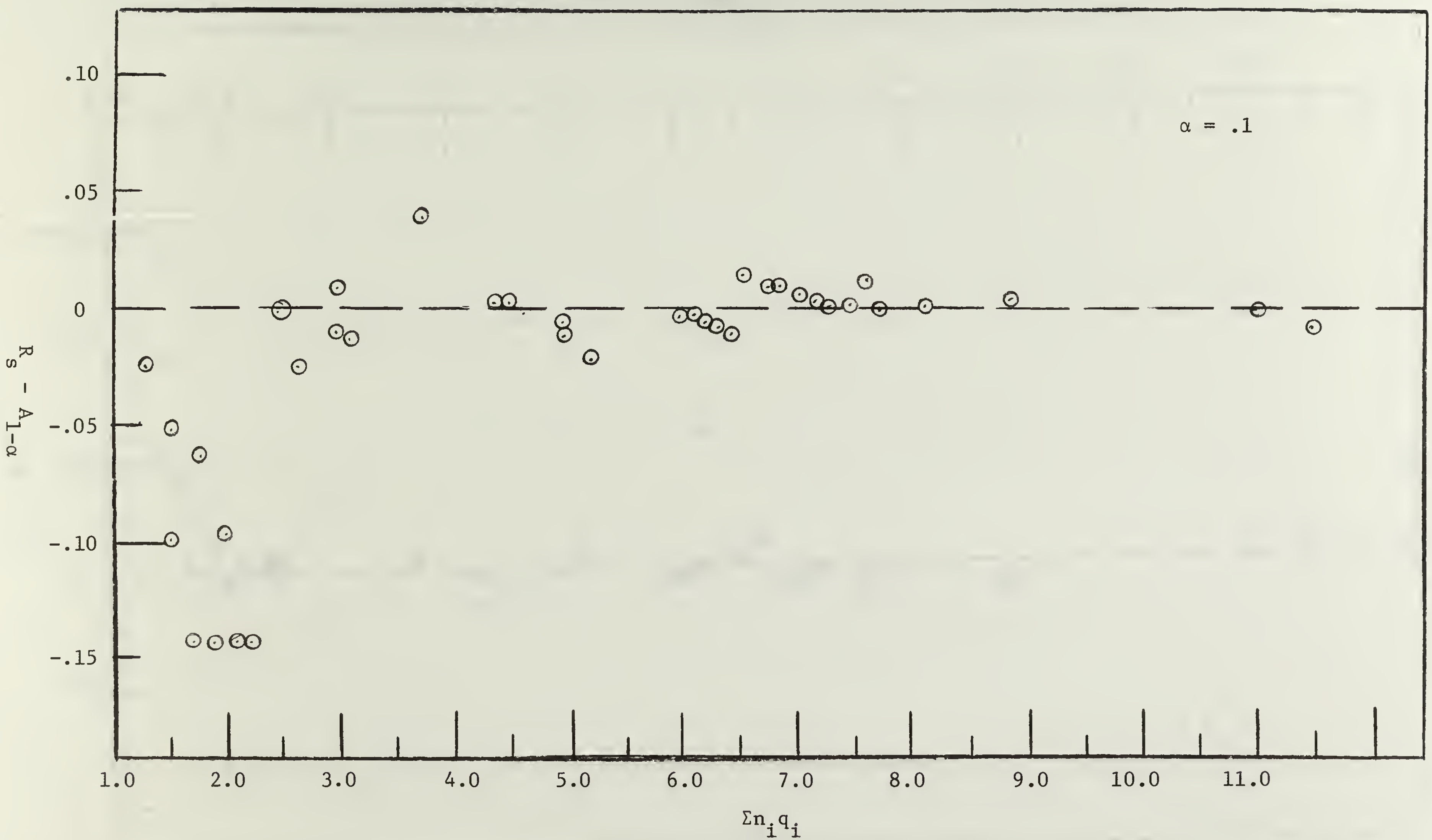

FIGURE 4

MEASURE OF THE ACCURACY CRITERION ($R_s - A_{1-\alpha}$) VERSUS $\Sigma n_i q_i$

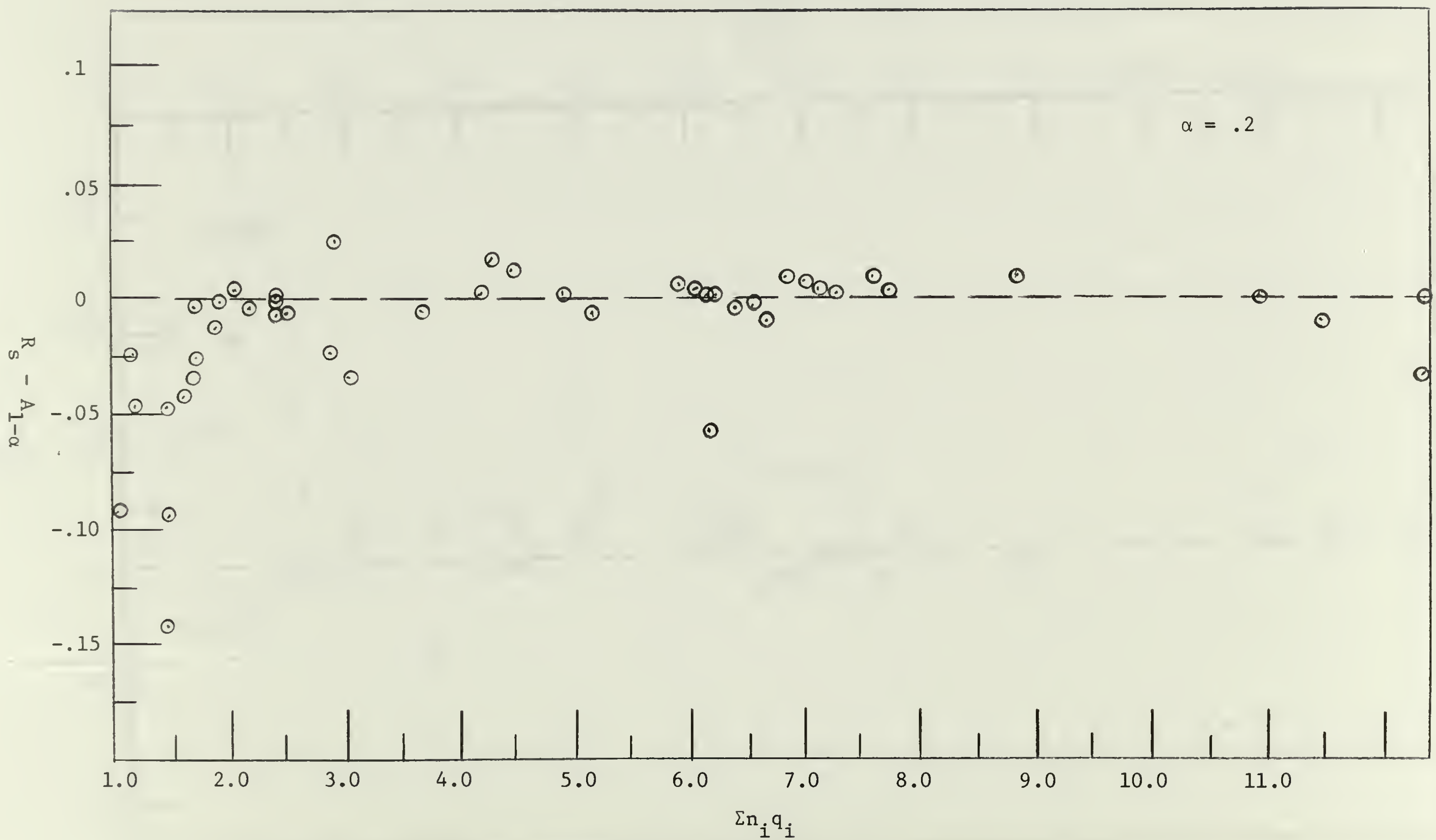

FIGURE 5

MEASURE OF THE ACCURACY CRITERION ($R_s - A_{1-\alpha}$) VERSUS $\Sigma n_i q_i$

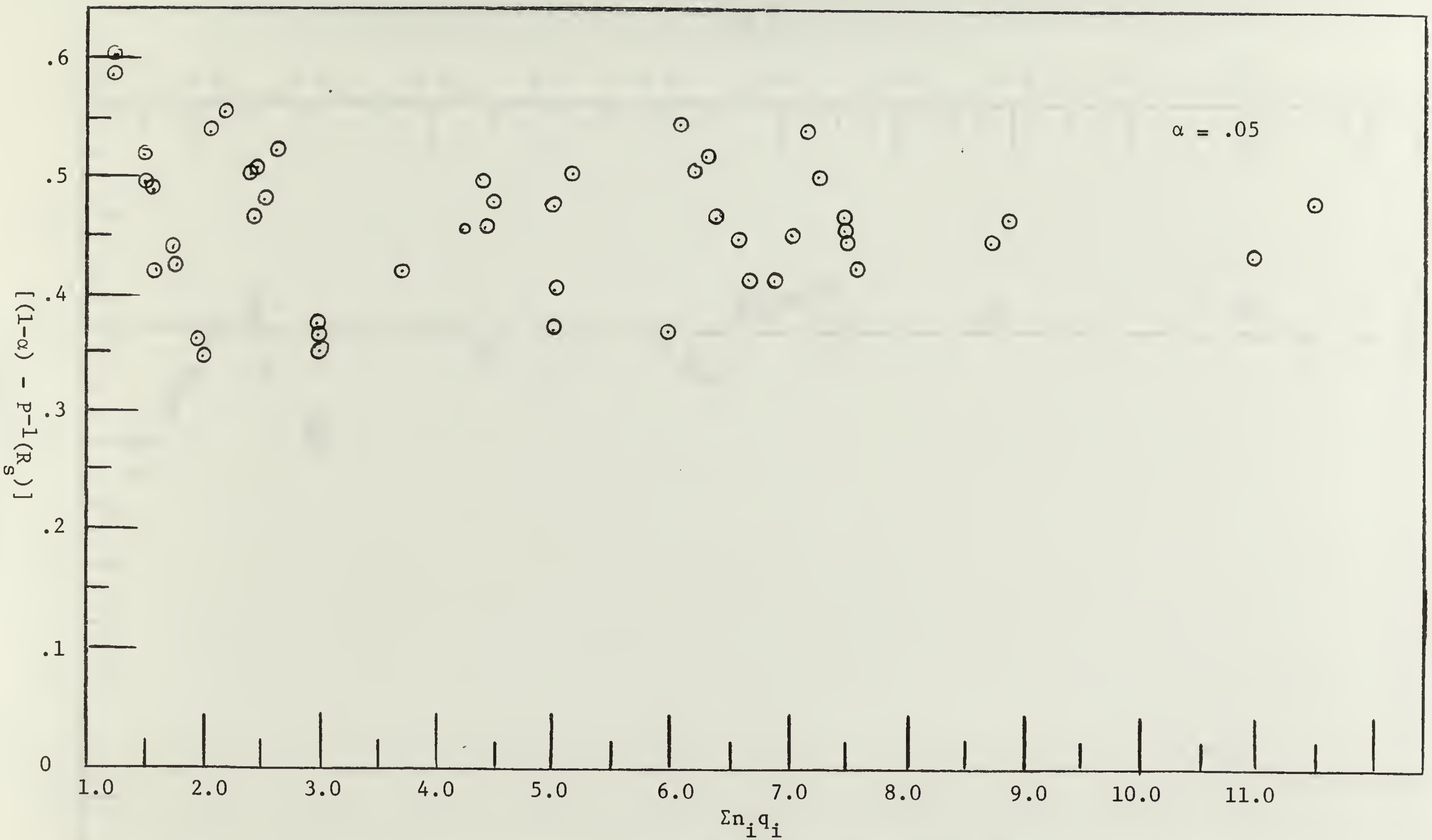

FIGURE 6

MEASURE OF THE ACCURACY CRITERION $[(1-\alpha) - P(R_s)]$ VERSUS $\Sigma n_i q_i$

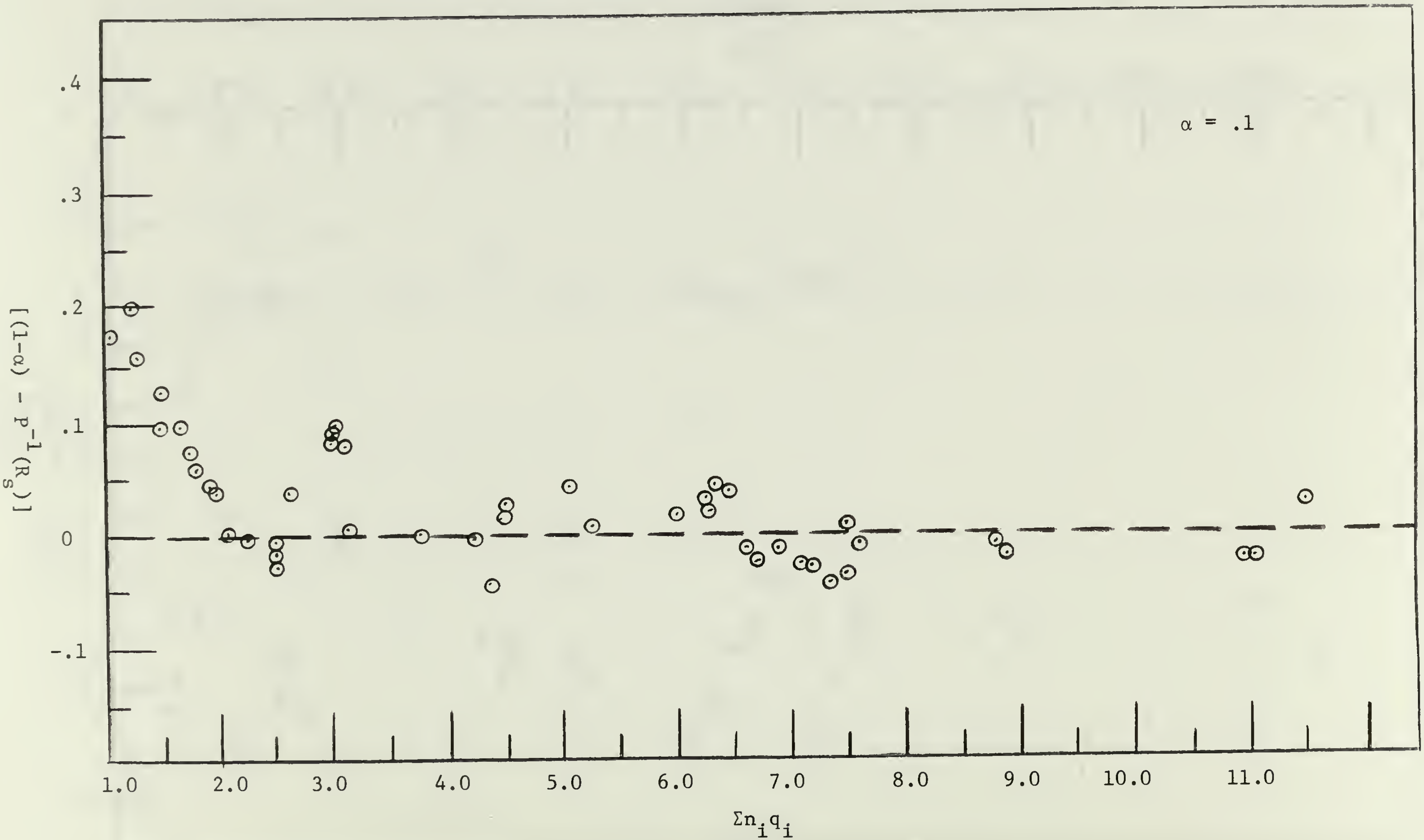

FIGURE 7

MEASURE OF THE ACCURACY CRITERION $[(1-\alpha) - P(R_s)]$ VERSUS $\Sigma n_i q_i$

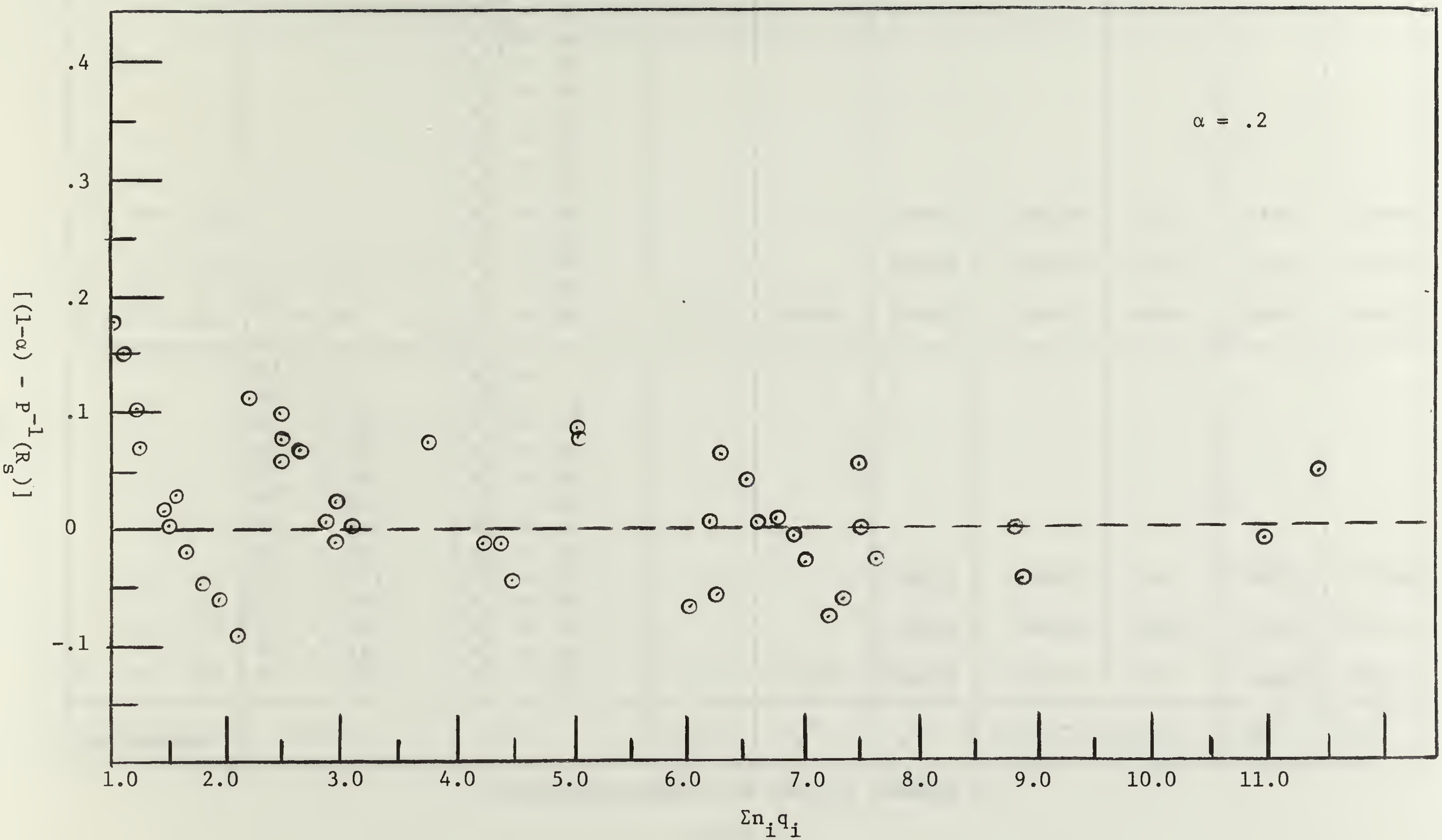

FIGURE 8

MEASURE OF THE ACCURACY CRITERION $[(1-\alpha) - P(R_s)]$ VERSUS $\Sigma n_i q_i$

TABLE I

SIMULATION RESULTS FOR CASES 1 THROUGH 24

CASE NUMBER	q_i	n_i	$\Sigma n_i q_i$	R_s	$A_{1-\alpha}$	$R_s - A_{1-\alpha}$	$P(R_s)$	$\mu_{\hat{R}_L}$	$\sigma_{\hat{R}_L}$
1 $\alpha = .05$	$q_1 = .005$ $q_2 = .015$ $q_3 = .021$ $q_4 = .012$ $q_5 = .018$ $q_6 = .02$ $q_7 = .003$ $q_8 = .005$ $q_9 = .03$ $q_{10} = .005$ $q_{11} = .032$ $q_{12} = .02$ $q_{13} = .1$	$n_1 = 150$ $n_2 = 90$ $n_3 = 75$ $n_4 = 100$ $n_5 = 125$ $n_6 = 18$ $n_7 = 28$ $n_8 = 125$ $n_9 = 63$ $n_{10} = 125$ $n_{11} = 59$ $n_{12} = 5$ $n_{13} = 19$	15.43	.72331	.86142	-.13811	.461	.72483	.088
$\alpha = .10$					.72779	-.00448	.89300	.59045	.1227
$\alpha = .20$					.73379	-.01048	.770	.64107	.11093
2 $\alpha = .05$	$q_i = .01$ $i = 1,2,\ldots,15$	$n_1 = 250$ $n_2 = 40$ $n_3 = 120$ $n_4 = 15$ $n_5 = 130$ $n_6 = 65$ $n_7 = 70$ $n_8 = 75$ $n_9 = 100$ $n_{10} = 90$ $n_{11} = 60$ $n_{12} = 60$ $n_{13} = 20$ $n_{14} = 30$ $n_{15} = 30$	11.55	.86006	.93433	-.07427	.4680	.85872	.05014
$\alpha = .10$					.86759	-.00753	.8770	.77878	.07083
$\alpha = .20$					.86861	-.00856	.750	.8120	.06480

Table I (continued)

CASE NUMBER	q_i	n_i	$\Sigma n_i q_i$	R_s	$A_{1-\alpha}$	$R_s - A_{1-\alpha}$	$P(R_s)$	$\mu_{\hat{R}_L}$	$\sigma_{\hat{R}_L}$
3 $\alpha = .05$	$q_i = .005$	$n_i = 20$	4.40	.79239	.950	-.15761	.4540	.79305	.08963
$\alpha = .10$	$i = 1,2,\ldots 14$	$i = 1,2,\ldots,15$			.79207	.00032	.9430	.65585	.11241
$\alpha = .20$	$q_{15} = .15$				.77636	.01604	.8160	.70426	.10391
4 $\alpha = .05$	$q_i = .005$	$n_i = 40$	8.80	.79239	.900	-.10761	.5020	.79439	.06064
$\alpha = .10$	$i = 1,2,\ldots,14$	$i = 1,2,\ldots,15$			.79205	.00035	.9140	.69402	.07415
$\alpha = .20$	$q_{15} = .15$				.79493	-.00254	.7950	.73159	.06922
5 $\alpha = .05$	$q_i = .005$	$n_i = 50$	11.00	.79239	.88357	-.09117	.5160	.7893	.05816
$\alpha = .10$	$i = 1,2,\ldots,14$	$i = 1,2,\ldots,15$			.78521	.00718	.9250	.70359	.0637
$\alpha = .20$	$q_{15} = .15$				.78932	.00308	.8090	.73802	.0587
6 $\alpha = .05$	$q_i = .005$	$n_i = 100$	22.0	.72939	.85675	-.06435	.5060	.79237	.03993
$\alpha = .10$	$i = 1,2,\ldots,14$	$i = 1,2,\ldots,15$			.78657	.00582	.92900	.72918	.04313
$\alpha = .20$	$q_{15} = .15$				.78563	.00676	.8420	.74834	.04367
7 $\alpha = .05$	$q_i = .005$	$n_i = 150$	7.65	.79239	.92479	-.1324	.5220	.78733	.0840
$\alpha = .10$	$i = 1,2,\ldots,14$	$i = 1,2,\ldots,5$			.78111	.01129	.9110	.66055	.1068
$\alpha = .20$	$q_{15} = .15$	$n_i = 20$			.79996	-.00756	.7740	.70772	.10288
		$i = 6,\ldots,15$							
8 $\alpha = .05$	$q_i = .005$	$n_i = 20$	23.90	.79239	.880	-.08761	.48400	.79330	.05387
$\alpha = .10$	$i = 1,2,\ldots,14$	$i = 1,2,\ldots,14$			.81324	-.02084	.8430	.70764	.07529
$\alpha = .2$	$q_{15} = .15$	$n_{15} = 150$			.80882	-.01643	.7380	.74255	.07156

Table I (continued)

CASE NUMBER		q_i	n_i	$\Sigma n_i q_i$	R_s	$A_{1-\alpha}$	$R_s - A_{1-\alpha}$	$P(R_s)$	$\mu_{\hat{R}_L}$	$\sigma_{\hat{R}_L}$
9	$\alpha = .05$	$q_i = .01$	$n_i = 50$	2.5	.95099	1.000	-.04901	.46500	.95076	.03007
	$\alpha = .10$	$i = 1,2,\ldots,5$	$i = 1,2,\ldots,5$			.94710	.00389	.9090	.90733	.04760
	$\alpha = .20$					.95924	-.00825	.7290	.92093	.04088
10	$\alpha = .05$	$q_i = .01$	$n_i = 50$	5.0	.90438	.96040	-.05602	.5680	.90414	.04063
	$\alpha = .10$	$i = 1,2,\ldots,10$	$i = 1,2,\ldots,10$			.91481	-.01043	.8580	.83963	.05526
	$\alpha = .20$					.90666	-.00228	.71800	.86752	.04817
11	$\alpha = .05$	$q_i = .01$	$n_i = 50$	7.5	.86006	.94119	-.08113	.490	.85961	.04632
	$\alpha = .10$	$i = 1,2,\ldots,15$	$i = 1,2,\ldots,15$			.85977	.00028	.9000	.78163	.05902
	$\alpha = .20$					.86112	-.00106	.74600	.81401	.05549
12	$\alpha = .05$	$q_i = .05$	$n_i = 100$	25.0	.77378	.84040	-.06662	.5380	.77327	.03992
	$\alpha = .10$	$i = 1,2,\ldots,5$	$i = 1,2,\ldots,5$			.76298	.01080	.9390	.70770	.0444
	$\alpha = .20$					.76706	.00672	.8210	.73251	.04158
13	$\alpha = .05$	$q_i = .005$	$n_i = 25$	3.125	.88222	.960	-.07778	.3720	.88114	.06307
	$\alpha = .10$	$i = 1,2,\ldots,25$	$i = 1,2,\ldots,25$			.89420	-.01198	.8200	.78420	.09101
	$\alpha = .20$					.91848	-.03626	.79700	.82409	.08276
14	$\alpha = .05$	$q_i = .005$	$n_i = 50$	6.25	.88222	.94119	-.05897	.440	.88176	.04374
	$\alpha = .10$	$i = 1,2,\ldots,25$	$i = 1,2,\ldots,25$			.88647	-.00476	.86900	.81133	.05885
	$\alpha = .20$					.88330	-.00108	.7420	.83840	.04995

Table I (continued)

CASE NUMBER	q_i	n_i	$\Sigma n_i q_i$	R_s	$A_{1-\alpha}$	$R_s - A_{1-\alpha}$	$P(R_s)$	$\mu_{\hat{R}_L}$	$\sigma_{\hat{R}_L}$
15 $\alpha = .05$	$q_i = .005$	$n_i = 100$	12.50	.88222	.93206	-.04985	.4870	.88219	.03239
$\alpha = .10$	$i = 1,2,\ldots,25$	$i = 1,2,\ldots,25$			.87880	.00342	.9400	.83130	.03493
$\alpha = .20$					.88454	-.00232	.77700	.85177	.03467
16 $\alpha = .05$	$q_i = .005$	$n_i = 25$	6.25	.77831	.88474	-.10643	.440	.77749	.07735
$\alpha = .10$	$i = 1,2,\ldots,50$	$i = 1,2,\ldots,50$			.77979	-.00148	.8730	.65253	.09817
$\alpha = .20$					.77597	.00234	.8660	.70017	.08446
17 $\alpha = .05$	$q_i = .005$	$n_i = 50$	12.50	.77831	.86813	-.08982	.4870	.77843	.05719
$\alpha = .10$	$i = 1,2,\ldots,50$	$i = 1,2,\ldots,50$			.76789	.01042	.940	.6880	.05889
$\alpha = .20$					.78036	-.00205	.7770	.72433	.05934
18 $\alpha = .05$	$q_i = .005$	$n_i = 25$	12.50	.60577	.75145	-.14568	.4870	.60624	.08947
$\alpha = .10$	$i = 1,2,\ldots,100$	$i = 1,2,\ldots,100$			.57646	.02930	.9400	.46388	.08305
$\alpha = .20$					.57426	.03151	.8640	.51687	.09016
19 $\alpha = .05$	$q_i = .001$	$n_i = 25$	.6250	.97530	1.000	-.02470	.430	.95638	.05440
$\alpha = .10$	$i = 1,2,\ldots,25$	$i = 1,2,\ldots,25$			1.000	-.02470	.4470	.94434	.06557
$\alpha = .20$					1.000	-.02470	.4700	.97502	.03063
20 $\alpha = .05$	$q_i = .001$	$n_i = 50$	1.250	.97530	1.000	-.02470	.347	.97593	.02104
$\alpha = .10$	$i = 1,2,\ldots,25$	$i = 1,2,\ldots,25$			1.000	-.02470	.7020	.94582	.04269
$\alpha = .20$					1.000	-.02470	.7050	.95788	.03337

Table I (continued)

CASE NUMBER	q_i	n_i	$\Sigma n_i q_i$	R_s	$A_{1-\alpha}$	$R_s - A_{1-\alpha}$	$P(R_s)$	$\mu_{\hat{R}_L}$	$\sigma_{\hat{R}_L}$
21 $\alpha = .05$	$q_i = .001$	$n_i = 100$	2.50	.97530	1.000	.02470	.4440	.97551	.01576
$\alpha = .10$	$i = 1,2,\ldots,25$	$i = 1,2,\ldots,25$			.97355	.00175	.92100	.95206	.02414
$\alpha = .20$					.97962	-.00432	.7070	.9603	.02084
22 $\alpha = .05$	$q_i = .001$	$n_i = 25$	1.250	.95120	1.000	-.04880	.34700	.95244	.04112
$\alpha = .10$	$i = 1,2,\ldots,50$	$i = 1,2,\ldots,50$			1.000	-.04880	.7020	.89353	.08260
$\alpha = .20$					1.000	-.04880	.70500	.91708	.06488
23 $\alpha = .05$	$q_i = .001$	$n_i = 50$	2.50	.95120	1.000	- .04880	.4440	.95171	.03073
$\alpha = .10$	$i = 1,2,\ldots,50$	$i = 1,2,\ldots,50$			.94710	.00410	.92100	.90579	.04663
$\alpha = .20$					.95924	-.00804	.7070	.92191	.0403
24 $\alpha = .05$	$q_i = .001$	$n_i = 25$	2.50	.90478	1.000	-.09522	.4440	.90578	.05847
$\alpha = .10$	$i = 1,2,\ldots,100$	$i = 1,2,\ldots,100$			.89420	.01059	.9210	.81775	.08698
$\alpha = .20$					.91848	-.01369	.7070	.84875	.07533

TABLE II

SIMULATION RESULTS FOR CASES 25 THROUGH 51

CASE NUMBER	q_i	n_i	$\Sigma n_i q_i$	R_s	$A_{1-\alpha}$	$R_s - A_{1-\alpha}$	$P(R_s)$	$\mu_{\hat{R}_L}$	$\sigma_{\hat{R}_L}$
25 $\alpha = .05$	$q_1 = .05$	$n_1 = 41$	4.27	.8664	.96154	-.09514	.4900	.86517	.06186
$\alpha = .10$	$q_2 = .05$	$n_2 = 26$			.86531	.00109	.9090	.76788	.08466
$\alpha = .20$	$q_3 = .04$	$n_3 = 23$			.86368	.00272	.8180	.80288	.07501
26 $\alpha = .05$	$q_1 = .1$	$n_1 = 40$	8.90	.7290	.85598	-.12698	.4800	.73319	.07706
$\alpha = .10$	$q_2 = .1$	$n_2 = 26$			.72340	.0056	.9200	.60014	.0924
$\alpha = .20$	$q_3 = .1$	$n_3 = 23$			.71984	.00916	.8410	.64682	.08542
27 $\alpha = .05$	$q_1 = .05$	$n_1 = 40$	4.45	.85737	.95652	-.09915	.4920	.85785	.06459
$\alpha = .10$	$q_2 = .05$	$n_2 = 26$			.86424	-.00687	.8850	.75173	.0888
$\alpha = .20$	$q_3 = .05$	$n_3 = 23$			.86195	-.00458	.7880	.79357	.07963
28 $\alpha = .05$	$q_1 = .03$	$n_1 = 40$	2.670	.91267	1.000	-.08733	.4270	.91452	.05062
$\alpha = .10$	$q_2 = .03$	$n_2 = 26$			.93387	-.0212	.8600	.83343	.0802
$\alpha = .20$	$q_3 = .03$	$n_3 = 23$			.92161	-.00894	.74200	.86478	.06978
29 $\alpha = .05$	$q_1 = .02$	$n_1 = 40$	1.780	.94119	1.0000	-.05881	.5080	.94020	.04340
$\alpha = .10$	$q_2 = .02$	$n_2 = 26$			1.0000	-.05881	.82700	.87871	.0779
$\alpha = .20$	$q_3 = .02$	$n_3 = 23$			.94905	-.00786	.72800	.89828	.06415

Table II (continued)

CASE NUMBER		q_i	n_i	$\Sigma n_i q_i$	R_s	$A_{1-\alpha}$	$R_s - A_{1-\alpha}$	$P(R_s)$	$\mu_{\hat{R}_L}$	$\sigma_{\hat{R}_L}$
30	$\alpha = .05$	$q_1 = .01$	$n_1 = 40$	.8900	.97030	1.000	-.02970	.4170	.97106	.03256
	$\alpha = .10$	$q_2 = .01$	$n_2 = 26$			1.000	-.02970	.580	.93321	.0663
	$\alpha = .20$	$q_3 = .01$	$n_3 = 23$			1.000	-.02970	.590	.94705	.05223
31	$\alpha = .05$	$q_i = .05$	$n_i = 30$	7.500	.77378	.87310	-.09941	.4980	.77172	.07010
	$\alpha = .10$	$i = 1,2,\ldots,5$	$i = 1,2,\ldots,5$			.77240	.00138	.9370	.65242	.0842
	$\alpha = .20$					.77227	.00151	.8080	.69984	.08006
32	$\alpha = .05$	$q_i = .05$	$n_i = 50$	12.50	.77378	.8666	-.09288	.4900	.77270	.05771
	$\alpha = .10$	$i = 1,2,\ldots,5$	$i = 1,2,\ldots,5$			.76514	.00864	.9480	.67909	.0612
	$\alpha = .20$					.77624	-.00245	.7940	.71527	.06147
33	$\alpha = .05$	$q_i = .05$	$n_i = 100$	25.0	.77378	.84030	-.06652	.5200	.77420	.03956
	$\alpha = .10$	$i = 1,2,\ldots,5$	$i = 1,2,\ldots,5$			.76287	.01091	.9470	.70977	.0434
	$\alpha = .20$					.76706	.00672	.8190	.73132	.04116
34	$\alpha = .05$	$q_i = .01$	$n_i = 30$	1.50	.95099	1.000	-.04901	.4480	.94949	.03980
	$\alpha = .10$	$i = 1,2,\ldots,5$	$i = 1,2,\ldots,5$			1.000	-.04901	.7760	.89888	.06782
	$\alpha = .20$					1.000	-.04901	.7760	.91885	.05630
35	$\alpha = .05$	$q_i = .01$	$n_i = 50$	2.50	.95099	1.000	-.04901	.46700	.95068	.02950
	$\alpha = .10$	$i = 1,2,\ldots,5$	$i = 1,2,\ldots,5$			.94710	.00389	.9200	.90529	.0465
	$\alpha = .20$					.95924	-.00825	.7450	.92060	.03903

Table II (continued)

CASE NUMBER	q_i	n_i	$\Sigma n_i q_i$	R_s	$A_{1-\alpha}$	$R_s - A_{1-\alpha}$	$P(R_s)$	$\mu_{\hat{R}_L}$	$\sigma_{\hat{R}_L}$
36 $\alpha = .05$	$q_i = .01$	$n_i = 100$	5.0	.95099	.98010	-.02911	.5720	.95056	.02122
$\alpha = .10$	$i = 1,2,\ldots,5$	$i = 1,2,\ldots,5$			.95706	-.00607	.86700	.91807	.0284
$\alpha = .20$					.95268	-.00169	.71600	.92992	.02712
37 $\alpha = .05$	$q_i = .005$	$n_i = 30$	.75	.97525	1.0000	-.02475	.51400	.97617	.02833
$\alpha = .10$	$i = 1,2,\ldots,5$	$i = 1,2,\ldots,5$			1.0000	-.02475	.5300	.94383	.0569
$\alpha = .20$					1.0000	-.02475	.54200	.95298	.04832
38 $\alpha = .05$	$q_i = .005$	$n_i = 50$	1.25	.97525	1.0000	-.02475	.35300	.97580	.02066
$\alpha = .10$	$i = 1,2,\ldots,5$	$i = 1,2,\ldots,5$			1.0000	-.02475	.7420	.94389	.0405
$\alpha = .20$					1.0000	-.02475	.7360	.95469	.03413
39 $\alpha = .05$	$q_i = .005$	$n_i = 100$	2.5	.97525	1.0000	-.02475	.4380	.97609	.01480
$\alpha = .10$	$i = 1,2,\ldots,5$	$i = 1,2,\ldots,5$			.97355	.00170	.91700	.95161	.02408
$\alpha = .20$					.97965	-.00437	.7220	.96020	.02055
40 $\alpha = .05$	$q_i = .05$	$n_i = 30$	15.0	.59874	.73260	-.13387	.5560	.59609	.07857
$\alpha = .10$	$i = 1,2,\ldots,10$	$i = 1,2,\ldots,10$			.57917	.01957	.9330	.46941	.0824
$\alpha = .20$					.57779	.02094	.8450	.51465	.07786
41 $\alpha = .05$	$q_i = .05$	$n_i = 50$	25.0	.59874	.7020	-.10326	.5380	.59663	.06274
$\alpha = .10$	$i = 1,2,\ldots,10$	$i = 1,2,\ldots,10$			.57452	.02422	.9550	.49537	.0595
$\alpha = .20$					.58555	.01319	.84900	.53554	.05968

Table II (continued)

CASE NUMBER	q_i	n_i	$\Sigma n_i q_i$	R_s	$A_{1-\alpha}$	$R_s - A_{1-\alpha}$	$P(R_s)$	$\mu_{\hat{R}_L}$	$\sigma_{\hat{R}_L}$
42 $\alpha = .05$	$q_i = .01$	$n_i = 30$	3.0	.90438	.96667	-.06229	.5840	.90207	.05318
$\alpha = .10$	$i = 1,2,\ldots,10$	$i = 1,2,\ldots,10$			.91183	-.00745	.8080	.82122	.0767
$\alpha = .20$					.93207	-.02768	.7830	.85417	.02768
43 $\alpha = .05$	$q_i = .01$	$n_i = 50$	5.0	.90438	.96040	-.05602	.5410	.90495	.04032
$\alpha = .10$	$i = 1,2,\ldots,10$	$i = 1,2,\ldots,10$			.91481	-.01043	.8530	.84130	.0555
$\alpha = .20$					.90666	-.00228	.7220	.86364	.05018
44 $\alpha = .05$	$q_i = .005$	$n_i = 30$	1.5	.95111	1.000	-.04889	.42800	.95238	.03699
$\alpha = .10$	$i = 1,2,\ldots,10$	$i = 1,2,\ldots,10$			1.000	-.04889	.7720	.89681	.0702
$\alpha = .20$					1.000	-.04889	.76100	.91964	.04889
45 $\alpha = .05$	$q_i = .005$	$n_i = 50$	2.50	.95111	1.0000	-.04889	.4470	.95231	.02893
$\alpha = .10$	$i = 1,2,\ldots,10$	$i = 1,2,\ldots,10$			.94710	.00401	.9130	.90522	.0479
$\alpha = .20$					.95924	-.00813	.72800	.92226	.0404
46 $\alpha = .05$	$q_i = .02$	$n_i = 5$	.5	.90392	1.0000	-.09608	.3990	.90476	.12615
$\alpha = .10$	$i = 1,2,\ldots,5$	$i = 1,2,\ldots,5$			1.0000	-.09608	.3870	.77651	.2875
$\alpha = .20$					1.0000	-.09608	.4010	.8175	.23079
47 $\alpha = .05$	$q_i = .02$	$n_i = 10$	1.0	.90392	1.0000	-.09608	.6340	.90343	.09327
$\alpha = .10$	$i = 1,2,\ldots,5$	$i = 1,2,\ldots,5$			1.0000	-.09608	.6420	.78644	.176
$\alpha = .20$					1.0000	-.09608	.62400	.82837	.150

Table II (continued)

CASE NUMBER	q_i	n_i	$\Sigma n_i q_i$	R_s	$A_{1-\alpha}$	$R_s - A_{1-\alpha}$	$P(R_s)$	$\mu_{\hat{R}_L}$	$\sigma_{\hat{R}_L}$
48 $\alpha = .05$	$q_i = .02$	$n_i = 15$	1.5	.90392	1.0000	-.09608	.4510	.9025	.07652
$\alpha = .10$	$i = 1,2,\ldots,5$	$i = 1,2,\ldots,5$			1.0000	-.09608	.807	.78892	.01307
$\alpha = .20$					1.0000	-.09608	.7710	.84026	.11135
49 $\alpha = .05$	$q_i = .02$	$n_i = 20$	2.0	.90392	1.0000	-.09608	.5990	.90542	.06251
$\alpha = .10$	$i = 1,2,\ldots,5$	$i = 1,2,\ldots,5$			1.0000	-.09608	.8640	.81014	.01029
$\alpha = .20$					.89810	.00582	.8630	.84586	.08994
50 $\alpha = .05$	$q_i = .02$	$n_i = 25$	2.50	.90392	1.0000	-.09608	.4790	.90320	.05710
$\alpha = .10$	$i = 1,2,\ldots,5$	$i = 1,2,\ldots,5$			.89420	.00972	.9250	.8157	.0868
$\alpha = .20$					.91848	-.01456	.7070	.84959	.07741
51 $\alpha = .05$	$q_i = .02$	$n_i = 30$	3.0	.90392	.96667	-.06275	.570	.90397	.05474
$\alpha = .10$	$i = 1,2,\ldots,5$	$i = 1,2,\ldots,5$			.91183	-.00791	.8150	.81856	.07757
$\alpha = .20$					.88716	.01676	.8020	.84915	.06861

TABLE III

SIMULATION RESULTS FOR CASES 52 THROUGH 73

CASE NUMBER	q_i	n_i	$\Sigma n_i q_i$	R_s	$A_{1-\alpha}$	$R_s - A_{1-\alpha}$	$P(R_s)$	$\mu_{\hat{R}_L}$	$\sigma_{\hat{R}_L}$
52 $\alpha = .05$	$q_i = .05$	$n_i = 5$	.75	.85737	1.0000	-.14263	.5490	.85582	.15246
$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			1.0000	-.14263	.5490	.66851	.3140
$\alpha = .20$					1.0000	-.14263	.5310	.74061	.25978
53 $\alpha = .05$	$q_i = .05$	$n_i = 10$	1.5	.85737	1.0000	-.14263	.4560	.86076	.10375
$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			1.0000	-.14263	.7770	.70324	.1895
$\alpha = .20$					1.0000	-.14263	.7840	.76032	.15697
54 $\alpha = .05$	$q_i = .05$	$n_i = 11$	1.65	.85737	1.0000	-.14263	.5240	.85370	.10233
$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			1.0000	-.14263	.8060	.70654	.1780
$\alpha = .20$					.81473	.04265	.8260	.75394	.15007
55 $\alpha = .05$	$q_i = .05$	$n_i = 12$	1.80	.85737	1.0000	-.14263	.51900	.86228	.09521
$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			1.0000	-.14263	.84600	.71323	.1570
$\alpha = .20$					.83017	.02721	.84900	.76715	.13232
56 $\alpha = .05$	$q_i = .05$	$n_i = 13$	1.95	.85737	1.0000	-.14263	.5840	.85759	.09345
$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			1.0000	-.14263	.8590	.71616	.1550
$\alpha = .20$					.84323	.01414	.865	.77046	.1280

Table III (continued)

CASE NUMBER	q_i	n_i	$\Sigma n_i q_i$	R_s	$A_{1-\alpha}$	$R_s - A_{1-\alpha}$	$P(R_s)$	$\mu_{\hat{R}_L}$	$\sigma_{\hat{R}_L}$
57 $\alpha = .05$	$q_i = .05$	$n_i = 14$	2.100	.85737	1.0000	-.14263	.4060	.86265	.09187
$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			1.0000	-.14263	.8910	.70868	.1450
$\alpha = .20$					.85443	.00295	.8910	.77051	.12144
58 $\alpha = .05$	$q_i = .05$	$n_i = 15$	2.250	.85737	1.0000	-.14263	.3860	.85663	.08831
$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			.82367	.03370	.9000	.72411	.1342
$\alpha = .20$					.86413	-.00676	.6900	.76565	.11535
59 $\alpha = .05$	$q_i = .05$	$n_i = 20$	3.00	.85737	.9500	-.09263	.5120	.85636	.07924
$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			.86775	.01038	.8220	.73029	.1098
$\alpha = .20$					.83282	.02455	.8180	.77710	.09524
60 $\alpha = .05$	$q_i = .05$	$n_i = 25$	3.75	.85737	.9600	-.10263	.5260	.85884	.06624
$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			.89420	.03683	.8990	.75165	.0890
$\alpha = .20$					.86525	-.00788	.7250	.79207	.08145
61 $\alpha = .05$	$q_i = .05$	$n_i = 30$	4.50	.85737	.96667	-.10929	.4660	.85908	.06071
$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			.85950	-.00213	.8680	.75990	.0817
$\alpha = .20$					.84565	.01173	.8500	.79305	.07412
62 $\alpha = .05$	$q_i = .05$	$n_i = 35$	5.250	.85737	.94367	-.08630	.4410	.85615	.05628
$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			.87737	-.0200	.8930	.76684	.0760
$\alpha = .20$					.86701	-.00964	.7930	.79653	.06772

Table III (continued)

CASE NUMBER	q_i	n_i	$\Sigma n_i q_i$	R_s	$A_{1-\alpha}$	$R_s - A_{1-\alpha}$	$P(R_s)$	$\mu_{\hat{R}_L}$	$\sigma_{\hat{R}_L}$
63 $\alpha = .05$	$q_i = .05$	$n_i = 40$	6.00	.85737	.9500	-.09263	.5770	.85585	.05242
$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			.8580	-.00063	.8820	.76698	.0678
$\alpha = .20$					.85385	.00353	.8660	.80098	.06109
64 $\alpha = .05$	$q_i = .05$	$n_i = 41$	6.15	.85737	.95122	-.09384	.4070	.85937	.05267
$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			.86137	-.004	.8770	.77062	.0673
$\alpha = .20$					.85811	-.00074	.7880	.80399	.06231
65 $\alpha = .05$	$q_i = .05$	$n_i = 42$	6.30	.85737	.95238	-.09501	.4350	.85948	.05365
$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			.86457	-.00720	.8790	.77259	.0653
$\alpha = .20$					.86135	-.00397	.7400	.80719	.06001
66 $\alpha = .05$	$q_i = .05$	$n_i = 43$	6.450	.85737	.93131	-.07394	.4840	.85617	.05065
$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			.86764	-.01024	.8650	.7777	.0672
$\alpha = .20$					.86367	-.00630	.7640	.80512	.06232
67 $\alpha = .05$	$q_i = .05$	$n_i = 44$	6.6	.85737	.93336	-.07598	.4960	.85775	.05146
$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			.84151	.01586	.9120	.7697	.06452
$\alpha = .20$					.86358	-.00621	.7990	.80373	.05960
68 $\alpha = .05$	$q_i = .05$	$n_i = 45$	6.75	.85737	.93432	-.07695	.5340	.85405	.04820
$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			.84487	.01250	.9270	.77314	.0615
$\alpha = .20$					.86878	-.01140	.7940	.80613	.05987

Table III (continued)

CASE NUMBER		q_i	n_i	$\Sigma n_i q_i$	R_s	$A_{1-\alpha}$	$R_s - A_{1-\alpha}$	$P(R_s)$	$\mu_{\hat{R}_L}$	$\sigma_{\hat{R}_L}$
69	$\alpha = .05$	$q_i = .05$	$n_i = 46$	6.90	.85737	.93573	-.07835	.5350	.85740	.05161
	$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			.84810	.00927	.9130	.77464	.063
	$\alpha = .20$					.84848	-.00890	.8140	.80544	.06012
70	$\alpha = .05$	$q_i = .05$	$n_i = 47$	7.050	.85737	.93708	-.07970	.4970	.85563	.0504
	$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			.85120	.00617	.9260	.77991	.0583
	$\alpha = .20$					.85155	.00583	.8270	.80888	.05731
71	$\alpha = .05$	$q_i = .05$	$n_i = 48$	7.20	.85737	.93837	-.08099	.4130	.85986	.04982
	$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			.85417	.00320	.92900	.77725	.0588
	$\alpha = .20$					.85330	.00407	.8730	.80447	.05513
72	$\alpha = .05$	$q_i = .05$	$n_i = 49$	7.35	.85737	.93961	-.08223	.4500	.85830	.04794
	$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			.85632	.00105	.9450	.77642	.0585
	$\alpha = .20$					.85674	.00064	.86900	.80802	.05448
73	$\alpha = .05$	$q_i = .05$	$n_i = 50$	7.5	.85737	.92198	-.06461	.4820	.85591	.04768
	$\alpha = .10$	$i = 1,2,3$	$i = 1,2,3$			.85838	-.00101	.8890	.77705	.0589
	$\alpha = .20$					.85780	-.00043	.7880	.80677	.05469

CHAPTER V

CONCLUSIONS

Based upon the results of the simulation, which are tabulated in Tables I through III and illustrated in Figures 3 through 8, the following general conclusions can be drawn:

a) The accuracy of the method is a function of $\Sigma n_i q_i$ and, in general, will increase as this value increases. However, the graphs of the accuracy criteria take the shape of a sawtooth curve. Therefore, there is no guarantee that increasing $\Sigma n_i q_i$ will improve the accuracy of the method. It is generally true, however, that as $\Sigma n_i q_i$ gets large the difference between system reliability, R_s, and $A_{1-\alpha}$ should converge to zero, as should the difference between the desired confidence level, $(1-\alpha)$, and $P(R_s)$.

b) At confidence levels of .9 and .8, and for values of $\Sigma n_i q_i$ which are sufficiently high, the procedure appears to be reasonably accurate. Obviously, the numerical value of "sufficiently high" is subjective and depends upon such unknown parameters as user risk, utility, etc. For most purposes, a value of $\Sigma n_i q_i \approx 4.0$ should be sufficiently high at both of the above confidence levels. The procedure appears to be inaccurate if R_s is very high or $\Sigma n_i q_i$ is small enough to make a failure only moderately probable.

c) Serious doubt exists as to the acceptability of the method for a confidence level of .95 or greater. At the former two confidence levels there was an apparent convergence toward zero error. No such convergence was apparent at the .95 level.

Even in Cases 6 and 8, where $\Sigma n_i q_i$ was 22.0 and 23.9, respectively, the difference, $R_s - A_{1-\alpha}$, is an order of magnitude greater than the corresponding values for $\alpha = .1$ and $\alpha = .2$. Similarly, the actual confidence level is about half of that intended.

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APPENDIX I

GLOSSARY

Meaning	Notation	Computer Simulation Variable Name
System reliability	R_s	RSUBS
Phase reliability	R_i	RSUBI(I)
System reliability estimator	$\hat{R}_s$	RHATS
Phase reliability estimator	$\hat{R}_i$	RHATI
Successful tests of the i^{th} phase	s_i	S(I)
Number of tests of the i^{th} phase	n_i	AN(I)
Estimator for the variance of the reliability estimators	$\hat{\sigma}_{\hat{R}_s}^2$	SIGPSQ
Estimator for the standard deviation of reliability estimators	$\hat{\sigma}_{\hat{R}_s}$	SIG
Computed lower confidence limit	$\hat{R}_L$	RHATL
The $(1-\alpha)^{th}$ percentile of the computed $\hat{R}_L$	$A_{1-\alpha}$	A1A
Probability that $\hat{R}_L$ is less than R_s	$P(R_s)$	RSPCN
The $(i)^{th}$ percentile of the standard normal distribution	$P(i)$	P(I)
Mean of the LCL's generated	$\mu_{\hat{R}_L}$	MEAN
Variance of the LCL's generated	$\sigma_{\hat{R}_L}^2$	VAR
Standard deviation of the LCL's generated	$\sigma_{\hat{R}_L}$	SIGMA
Number of replications per case	KK	KK
Number of phases per case	K	K

APPENDIX II

PROGRAM PRINTOUT

```
      PROGRAM MAIN
C
C
C     PROGRAM MAIN DOES ALL COMPUTATIONS EXCEPT
C     THE COMPUTATION OF THE VALUES RHATS, SYSTEM
C     RELIABILITY ESTIMATOR, AND RHATL, LOWER
C     CONFIDENCE LIMITS.  THESE LATTER COMPUTA-
C     TIONS ARE DONE IN SUBROUTINE RHAT
C
C
C
C
C     HERE SET INDEX OF RANDOM NUMBER GENERATOR
C
      XXX=RNG(0)
      XXX=RNG(151)
C
C
C     HERE SET ALL SUBSYSTEM PARAMETERS AND TEST
C     STATISTICS TO ZERO TO AVOID CARRYING VALUES
C     OVER FROM ONE CASE STUDY TO THE NEXT.
C
      DO 9 J=1,100
      R(J)=0.
      AN(J) =0.
      S(J)=0.
    9 CONTINUE
C
C
C     HERE READ INTO MEMORY:  NUMBER OF REPLI-
C     CATIONS DESIRED;  NUMBER OF SUBSYSTEMS CON-
C     SIDERED;  ALFA LEVEL;  RELIABILITY AND NUM-
C     BER OF TESTS FOR EACH SUBSYSTEM.
C
      READ(5,900)KK,K,ALFA,(  R(J),AN(J), J=1,K)
  900 FORMAT (2I10, F10.5,/( 2F10.5))
C
C
C     HERE READ INTO MEMORY TEN VALUES OF STAND-
C     ARD DEVIATION OF (1-ALFA) PERCENTILE OF A
C     STANDARD NORMAL DISTRIBUTION.
C
      P(1)=1.645
      P(2)=1.2816
      P(3)=1.038
      P(4)=.8416
      P(5)=.675
      P(6)=.525
      P(7)=.385
      P(8)=.253
      P(9)=.124
      P(10)=.0
      RK=KK
C
C
C     HERE INITILIZE THE VALUES OF ALL CUMULA-
C     TIVE SUMS AND PRODUCTS TO ZERO AND ONE,
C     RESPECTIVELY.
C
      RSUBS=1.
      DO 20 J=1,KK
      RHATL(J) =0.
   20 CONTINUE
      PMU= 0.
      SUM= 0.
      SUMA = 0.
C
```

```
C         HERE COMPUTE SYSTEM RELIABILITY, THAT IS
C         THE PRODUCT OF ALL SUBSYSTEM RELIABILITIES.
C
      DO 50 IZ=1,K
      RSUBS=RSUBS*R(IZ)
   50 CONTINUE
C
C
C         HERE COMPUTE THE SUM OF PRODUCTS
C         ( N(I) X Q(I) ), THE AMOUNT OF TESTING
C         TIMES THE SUBSYSTEM UNRELIABILITY, FOR
C         ALL SUBSYSTEMS.
C
      DO 450 JR=1,K
      SUMA = (AN(JR)*(1.-  R(JR)))+SUMA
  450 CONTINUE
C
C
C         HERE BEGIN GATHERING TEST DATA FOR EACH
C         OF KK REPLICATIONS.
C
      DO 100 IX=1,KK
C
C
C         HERE SET THE VALUE S(4), THE NUMBER OF
C         SUCCESSFUL TRIALS, TO ZERO FOR EACH SUB-
C         SYSTEM.  BEGIN TESTING.
C
      DO 10 J=1,K
      S(J)=0.
   10 CONTINUE
      DO 200 IA=1,K
      NIA=AN(IA)
      DO 200 IB=1,NIA
C
C
C         HERE COMPARE AN(IA) RANDOM NUMBERS TO
C         RELIABILITY OF THE (IA)TH SUBSYSTEM,
C         AN(IA) BEING THE NUMBER OF TESTS DESIRED
C         FOR THE (IA)TH SUBSYSTEM.  IF THE RANDOM
C         NUMBER IS EQUAL TO OR LESS THAN SUBSYSTEM
C         RELIABILITY A SUCCESSFUL TEST IS COUNTED.
C
      IF (  R(IA)-RNG(1)) 200,199,199
  199 S(IA)=S(IA)+1.
  200 CONTINUE
C
C
C         HERE CALL SUBROUTINE 'RHAT' WHICH COMPUTES
C         THE LOWER CONFIDENCE LIMIT  FOR THE TEST
C         STATISTICS GATHERED ABOVE.  THE LOWER CON-
C         FIDENCE LIMIT IS RETURNED AND STORED IN
C         TWO DIFFERENT VECTOR MATRICES 'RHATL' AND
C         'A1' FOR FURTHER COMPUTATIONS.  THEY ARE
C         ALSO SUMMED FOR LATER COMPUTATION OF THEIR MEAN
C         SUBROUTINE 'RHAT' ALSO RETURNS KK VALUES
C         OF THE SYSTEM RELIABILITY ESTIMATOR 'RHATS'.
C         THESE VALUES ARE ALSO STORED IN A VECTOR
C         MATRIX FOR LATER PRINTOUT.
C
      CALL RHAT(ALFA,PHIH,CAPA)
      RHATL(IX)=PHIH
      RHATS(IX)=CAPA
      A1(IX)=RHATL(IX)
      PMU=PMU+RHATL(IX)
  100 CONTINUE
C
C
```

```
C         HERE THE LOWER CONFIDENCE LIMIT MATRIX AND
C         THE SYSTEM RELIABILITY ESTIMATOR MATRIX
C         ARE SORTED INTO ASCENDING ORDER OF THEIR
C         VALUES.
C
      CALL SHSORT(A1,A2,KK)
      CALL SHSORT(RHATS,CAP1,KK)
C
C
C         HERE DIVIDE ACCUMULATED SUMS OF LOWER CON-
C         FIDENCE LIMITS BY THE NUMBER OF REPLICA-
C         TIONS TO COMPUTE THEIR MEAN.
C
      MEAN=PMU/BK
C
C
C         HERE COMPUTE THE VALUE 'KALFA' THAT IS
C         ( 1 - ALFA ) X THE NUMBER OF REPLICATIONS.
C         THIS VALUE IS USED AS THE INDEX NUMBER OF
C         THE  ORDERED LOWER CONFIDENCE LIMIT MATRIX.
C         THUS A1(KALFA) IS THE (1-ALFA)TH PERCENTILE
C         OF THE LOWER CONFIDENCE LIMIT DISTRIBUTION.
C
      AALFA=(1.-ALFA)*BK
      KALFA=AALFA
      A1A=A1(KALFA)
C
C
C         HERE COMPUTE THE PERCENTILE ASSOCIATED WITH
C         R SUB S IN A WAY SIMILAR TO THE METHOD
C         DESCRIBED ABOVE.
C
      RST=RSUBS
      DO 400 IC=1,KK
      IF(RST-A1(IC)) 398,399,399
  398 KIC=IC-1
      PIC=KIC
      RSPCN =PIC/BK
      RST=1.1
C
C
C         HERE COMPUTE VARIANCE AND STANDARD DEVI-
C         ATION OF THE LOWER CONFIDENCE LIMIT DISTRIBUTION.
C
  399 NUM=RHATL(IC)-MEAN
      NUMSQ=NUM*NUM
      SUM=SUM+NUMSQ
  400 CONTINUE
      VAR = SUM/BK
      SIGMA = SQRT(VAR)
C
C
C         HERE COMPUTE THE VALUE RSUBS - A SUB(1-ALFA).
C
      DIFF = RSUBS-A1A
C
C
C         HERE PRINT OUT ALL DESIRED VALUES AND RESULTS.
C
      WRITE(6,1010)
      WRITE(6,1011)(J,AN(J),  R(J),J=1,K)
      WRITE(6,1002)RSUBS
      WRITE(6,1003)A1A
      WRITE(6,1004)SIGMA
      WRITE(6,1005)RSPCN
      WRITE(6,1006)MEAN
      WRITE(6,1023)DIFF
      WRITE(6,1024)SUMA
 1023 FORMAT(/////10X,'R SUB S - A SUB(1-ALFA) EQUALS',
     *F15.5)
 1024 FORMAT(/////10X,'SIGMA (N(I) X Q(I)) EQUALS',F15.5)
 1010 FORMAT(1H1,38X,'I',12X,'N(I)', 9X,'R(I)'//)
```

```
1011 FORMAT(38X,I2,F15.0,F15.3)
1001 FORMAT (/////(10X,5F15.5))
1002 FORMAT (/////10X,'R SUB S (SYSTEM RELIABILITY)',
    *' EQUALS',F15.5)
1003 FORMAT (/////10X,'(1-ALFA) X 100 PERCENTILE ',
    *'(A SUB 1 - ALFA) EQUALS ',F15.5)
1004 FORMAT (/////10X,'STANDARD DEVIATION OF COMPUTED ',
    *'LOWER CONFIDENCE LIMIT EQUALS',F15.5)
1005 FORMAT (/////10X,'PERCENTILE ASSOCIATED WITH '
    *'R SUB S EQUALS',F15.5)
1006 FORMAT(////10X,'MEAN OF COMPUTED LOWER CONFIDENCE',
    *'LIMIT EQUALS',F15.5)
1020 FORMAT(10X,'1000 VALUES OF THE LOWER CONFIDENCE ',
    *'LIMIT, LISTED IN ACCENDING ORDER')
1021 FORMAT(10X,'1000 VALUES OF THE LOWER CONFIDENCE ',
    *'LIMIT,LISTED IN THE ORDER GENERATED')
1022 FORMAT(10X,'1000 VALUES OF THE ESTIMATOR FOR THE',
    *' SYSTEM RELIABILITY (RHATS)')
     WRITE(6,1020)
     WRITE(6,1001)A1
     WRITE(6,1021)
     WRITE(6,1001)RHATL
     WRITE(6,1022)
     WRITE(6,1001)RHATS
     STOP
     END
```

```
C                        SUBROUTINE RHAT
C
C
C
C         SUBROUTINE RHAT COMPUTES THE VALUES RHATS
C         (SYSTEM RELIABILITY ESTIMATOR) AND RHATL
C         (LOWER CONFIDENCE LIMIT) USING AS ITS
C         CALLING PARAMETERS THE STATISTICS GENERATED
C         IN PROGRAM MAIN.  IT IS A STRAIGHTFORWARD
C         APPLICATION OF THE APPLIED PHYSICS LAB-
C
C
C
      COMMON R(100),AN(100),S(100),RHATL(1000),P(10),
     *A1(1000),A2(1000),KK,K
C
C
C         HERE INITIALIZE VALUES AS NECESSARY
C
      PIPSQ=1.
      PROD=1.
      CAPP=1.
C
C
C         HERE 'RHATI' IS SYBSYSTEM RELIABILITY ESTIMATOR
C
      DO 500 JA=1,K
      RHATI=S(JA)/AN(JA)
      PSQ=RHATI*RHATI
C
C
C         HERE PIPSQ IS THE PRODUCT OF THE SQUARED
C         SUBSYSTEM ESTIMATORS.
C
      PIPSQ=PIPSQ*PSQ
C
C
C         THE PURPOSE OF THIS STATEMENT IS TO PRE-
C         VENT EXPONENT UNDERRUN, THAT IS, IT PRE-
C         VENTS NUMBERS FROM GETTING SMALLER THAN
C         THE COMPUTER CAN EFFECTIVELY HANDLE. IT
C         SETS VERY SMALL NUMBERS EQUAL TO ZERO.
C
      IF(PIPSQ-.0000001)501,502,502
  501 PIPSQ=0.
      PROD=0.
C
C
C         THIS STATEMENT IS A CODIFICATION OF THE
C         SECOND TERM OF THE RIGHT HAND SIDF OF
C         EQUATION (5), CHAPTER II.
C
  502 PROD=PROD*(PSQ-(RHATI*(1.-RHATI)) /(AN(JA)-1.))
      CAPP=CAPP*RHATI
  500 CONTINUE
C
C
C         THIS STATEMENT IS A CODIFICATION OF THE
C         ENTIRE EQUATION (5), CHAPTER II.
C
      SIGPSQ=PIPSQ-PROD
      SIGP=SQRT(SIGPSQ)
      BALFA=ALF *20.
      JALFA=BALFA
C
C
C         HERE COMPUTE THE VALUE OF THE LOWER CONFI-
C         DENCE LIMIT, CALLED PHA IN THIS SUBROUTINE.
C
      PHA =CAPP-(P(JALFA)*SIGP)
      RETURN
```

SUBROUTINE SHSORT

```
      SUBROUTINE SHSORT(A,KEY,/N/)
      DIMENSION A(N),KEY(N)
      M1=1
    6 M1=M1*2
      IF(M1-N) 6,6,8
    8 M1=M1/2-1
      MM=MAX0(M1/2,1)
      GO TO 21
   20 MM=MM/2
      IF(MM)100,100,21
   21 K=N-MM
   22 DO 1  J=1,K
      II=J
   11 IM=II+MM
      IF(A(IM)-A(II)) 30,1,1
   30 TEMP=A(II)
      IT=KEY(II)
      A(II)=A(IM)
      KEY(II)=KEY(IM)
      A(IM)=TEMP
      KEY(IM)=IT
      II=II-MM
      IF(II) 1,1,11
    1 CONTINUE
      GO TO 20
  100 RETURN
      END
```

FUNCTION SUBROUTINE RNG

```
      FUNCTION RNG(N)
      NR=N
      IF(NR)10,10,20
   10 IX=30517
      NR=NR+1
   20 DO 50 I=1,NR
      IY=IX*65539
      IF(IY)5,6,6
    5 IY=IY+2147483647+1
    6 RNG=IY
      RNG=RNG*.4656613E-9
   50 IX=IY
      RETURN
      END
```

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13. ABSTRACT

Systems which are composed of two or more phases, or subsystems, arranged in logical sequence are found frequently in industry and defense. Standard procedures for computing lower confidence limits on reliability of such systems rely on the use of system data. Engineering changes to any of these subsystems can effect the invalidation of all existing data, necessitating additional, sometimes extensive, testing. Such changes are not infrequent in complex systems. A need exists for a method of computing lower confidence limits on reliability which uses phase data. Some approximation techniques have become available. One such technique is currently being used by Applied Physics Laboratory, The Johns Hopkins University. Computer simulation techniques are used to analyze the accuracy of this procedure.

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14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Reliability						
System Reliability						
Multi-phase System						
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